

# Calculus of variations

## Euler-Lagrange equation

$$S(q) = \int_a^b L(t, q(t), q'(t)) dt$$

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•  $q$  :

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$$q : [a, b] \subset \mathbb{R} \rightarrow X \quad t \mapsto x = q(t)$$

$$q(a) = x_a, \quad q(b) = x_b$$

•  $q'$   $q$  .

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$$1 -$$

$$f(a) = c, \quad f(b) = d$$

$$J = \int_a^b F(x, f(x), f'(x)) dx$$

$$F(x, f(x), f'(x)) \text{ 가 } f(x) \text{ 가 } f'(x) \text{ . (가 , )}$$

$$f(x) \text{ , } f'(x) \text{ 가 , } J(x, f(x), f'(x))$$

$$f(x) + \epsilon \eta(x) \text{ , } g_\epsilon(x) = f(x) + \epsilon \eta(x) \text{ , } \eta(a) = \eta(b) = 0 \text{ , } g'_\epsilon(x) = f'(x) + \epsilon \eta'(x)$$

$$J(\epsilon) = \int_a^b F(x, g_\epsilon(x), g'_\epsilon(x)) dx$$

$$J(\epsilon) \text{ , } \epsilon$$

$$\frac{dJ}{d\epsilon}(0) = \int_a^b \left( \frac{\partial F}{\partial x} \eta + \frac{\partial F}{\partial f} \eta' \right) dx$$

$$\frac{dJ}{d\epsilon}(0) = \frac{\partial F}{\partial x} \eta + \frac{\partial F}{\partial f} \eta'$$

$$x \frac{\partial x}{\partial \epsilon} + \frac{\partial F}{\partial g_{\epsilon}} \frac{\partial g_{\epsilon}}{\partial \epsilon} + \frac{\partial F}{\partial g'_{\epsilon}} \frac{\partial g'_{\epsilon}}{\partial \epsilon} = \eta(x) \frac{\partial F}{\partial g_{\epsilon}} + \eta'(x) \frac{\partial F}{\partial g'_{\epsilon}}. \quad \text{end{equation}}$$

$$\begin{equation} \frac{dJ}{d\epsilon} = \int_a^b \left[ \eta(x) \frac{\partial F}{\partial g_{\epsilon}} + \eta'(x) \frac{\partial F}{\partial g'_{\epsilon}} \right] dx. \\ \text{end{equation}} \end{equation}$$

$$\epsilon = 0 \quad g_{\epsilon} = f, \quad f \text{ 가 } J \\ J'(0) = 0, \quad .$$

$$\begin{equation} J'(0) = \int_a^b \left[ \eta(x) \frac{\partial F}{\partial f} + \eta'(x) \frac{\partial F}{\partial f'} \right] dx = 0. \quad \text{end{equation}} \end{equation}$$

$$, \quad .$$

$$\begin{equation} 0 = \int_a^b \left[ \frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'} \right] \eta(x) dx + \left[ \eta(x) \frac{\partial F}{\partial f'} \right]_a^b. \\ \text{end{equation}} \end{equation}$$

$$\eta, \quad ,$$

$$\begin{equation} 0 = \int_a^b \left[ \frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'} \right] \eta(x) dx. \quad \text{end{equation}} \end{equation}$$

$$, \quad - \quad .$$

$$\begin{equation} 0 = \frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'}. \\ \text{end{equation}} \end{equation}$$

가

$$2 \quad \left( a, y_a \right) \quad \left( b, y_b \right) \text{가} \quad . \\ \text{가} \quad L$$

$$\begin{equation} L[f] = \int_a^b \sqrt{1 + f'(x)^2} dx \quad \text{end{equation}} \end{equation}$$

$$f(a) = y_a, \quad f(b) = y_b$$

$$- \quad , \quad f$$

$$\begin{equation} 0 = - \frac{d}{dx} \frac{\partial L}{\partial f'} \sqrt{1 + f'(x)^2} \\ \text{end{equation}} \end{equation}$$

$$, \quad ,$$

$$\begin{equation} \begin{matrix} 0 = & \frac{d}{dx} \frac{\partial L}{\partial f'} \sqrt{1 + f'(x)^2} \\ & \frac{d}{dx} \frac{f'(x)}{\sqrt{1 + f'(x)^2}} \end{matrix} \\ \text{end{matrix}} \quad \text{end{equation}} \end{equation}$$

0

,

$$\frac{f'(x)}{\sqrt{1 + f'(x)^2}} = k$$

가 ,

$$f'(x)$$

.

$$f(x) = C$$

가

$$f(x) = Cx + D$$

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