

Calculus of variations

Euler-Lagrange equation

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial q'} = 0$$

$$\int_a^b L(t, q(t), q'(t)) dt$$

$$q: [a, b] \rightarrow X$$

$$q|_{[a, b]} : [a, b] \rightarrow X$$

$$q(a) = x_a, q(b) = x_b$$

$$q' : [a, b] \rightarrow X$$

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$$f: [a, b] \rightarrow \mathbb{R}$$

$$J = \int_a^b F(x, f(x), f'(x)) dx$$

$$F(f) = \int_a^b f(x) dx$$

$$\begin{aligned} f(x) + \epsilon \eta(x) &= f(a) + \epsilon \eta(a) \\ f'(x) &= \eta'(x) \\ f(b) &= \eta(b) = 0 \end{aligned}$$

$$J(\epsilon) = \int_a^b F(x, f(x) + \epsilon \eta(x), f'(x) + \epsilon \eta'(x)) dx$$

$$J'(\epsilon) = \int_a^b \frac{\partial F}{\partial f} \eta(x) dx$$

$$\frac{\partial F}{\partial f} = \frac{1}{2} \int_a^b \frac{\partial^2 F}{\partial f^2} (x, f(x), f'(x)) dx$$

$$\frac{\partial F}{\partial f} = \frac{1}{2} \int_a^b \frac{\partial^2 F}{\partial f^2} (x, f(x), f'(x)) dx$$

$$x\}\frac{\partial x}{\partial \varepsilon} + \frac{\partial F}{\partial g}\frac{\partial g}{\partial \varepsilon} + \frac{\partial F}{\partial g'}\frac{\partial g'}{\partial \varepsilon} = \eta(x) \frac{\partial F}{\partial g} + \eta'(x) \frac{\partial F}{\partial g'}. \quad \text{end{equation}}$$

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\begin{equation} \frac{\mathrm{d} \, J}{\mathrm{d} \, \epsilon} = \int_a^b \left[ \eta(x) \frac{\partial F}{\partial g_\epsilon} + \eta'(x) \frac{\partial F}{\partial g_\epsilon'} \right] dx.
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$$\$epsilon = 0\$ \quad g_epsilon = f\$, f\$ 가 J\$ \\ J'left(0\right) = 0\$, . ,$$

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\begin{equation} J'(0) = \int_a^b \left[ \eta(x) \frac{\partial F}{\partial f} + \eta'(x) \frac{\partial F}{\partial f'} \right] dx = 0. \end{equation}
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\begin{equation} 0 = \int_a^b \left[ \frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'} \right] \eta(x) dx + \left[ \eta(x) \frac{\partial F}{\partial f'} \right]_a^b.
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$\$\\eta\$$

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\begin{equation} 0 = \int_a^b \left[ \frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'} \right] \eta(x) dx. \end{equation}
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\begin{equation} 0 = \frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'}.
\end{equation}
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가

$$2 \quad \left(a, y_a\right) \quad \left(b, y_b\right) \text{ 가} \\ \text{가} \quad \$L\$$$

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\begin{equation} L\left[f\right] = \int_a^b \sqrt{1 + f'\left(x\right)^2} dx \end{equation}
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$$f(a) = y_a, f(b) = y_b$$

\$f\$

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\begin{equation} 0 = -\frac{d}{dx}\frac{\partial}{\partial f'}\sqrt{1 + f'^2(x)} \end{equation}
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\begin{equation} \begin{matrix} 0 &=& \frac{d}{dx} \frac{\partial}{\partial f'} \sqrt{1 + f' \left( x \right)^2} \\ &=& \frac{d}{dx} \frac{f' \left( x \right)}{\sqrt{1 + f' \left( x \right)^2}} \end{matrix} \end{equation}
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0

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$$\begin{aligned} & \text{\backslash begin\{equation\} \backslash frac\{f'\backslash left(x\backslash right)\}\{\backslash sqrt\{1 + f'\backslash left(x\backslash right)^2\}\} = k \backslash end\{equation\}} \\ & \text{가 , } \end{aligned}$$

$$f'\backslash left(x\backslash right)$$

$$\begin{aligned} & \text{\backslash begin\{equation\} f'\backslash left(x\backslash right) = C \backslash end\{equation\}} \\ & \text{가 } \end{aligned}$$

$$f\backslash left(x\backslash right)=Cx + D$$

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