

Calculus of variations

Euler-Lagrange equation

-\$q\$ \$S\$ \$q\left(t\right)\$
\$S\$

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\begin{displaymath} \displaystyle S(q) = \int_a^b L(t,q(t),q'(t)), \mathrm{d}t \end{displaymath}
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- \$q\$

20

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\begin{displaymath} \begin{aligned} q \colon [a, b] \subset \mathbb{R} &\rightarrow X \\ t &\mapsto x = q(t) \end{aligned} \end{displaymath}
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$\$q\$$ 가 , $q\left(a\right) = x_a$, $q\left(b\right) = x_b$

- \$q'\$ \$q\$

$$f(a) = c, f(b) = d$$

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\begin{displaymath} J = \int_a^b F(x, f(x), f'(x)) \, dx. \end{displaymath}
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\$F\$가 가 가 . (가 .)

$\$f\$$ 가 , $\$f\$$ 가 , $\$J\$$
 $(\$f\$ 가 \$J\$) \$J\$. (\$f\$ 가 \$J\$)$

$\$f\$$ $\$g_epsilon\left(x\right) =$
 $\left.f\left(x\right)\right| + \epsilon\eta\left(x\right) . \eta\left(x\right) \eta\left(a\right) =$
 $\eta\left(b\right) = 0$ 가 . , $\$f\$$ $\$g\$$ $\$J\$$
 가

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\begin{displaymath} J(\epsilon) = \int_a^b F(x, g_\epsilon(x), g_\epsilon'(x)) dx. \end{displaymath}
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$\$| \$ \quad \$\backslash epsilon \$$

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\begin{displaymath} \frac{\mathrm{d}}{\mathrm{d} \varepsilon} \int_a^b F(\mathrm{d} \varepsilon) \epsilon(x, g_\varepsilon(x), g_\varepsilon'(x)) \, dx. \end{displaymath}
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\begin{displaymath} \frac{\partial F}{\partial \epsilon} = \frac{\partial F}{\partial x} \frac{\partial x}{\partial \epsilon} + \frac{\partial F}{\partial g} \frac{\partial g}{\partial \epsilon} + \frac{\partial F}{\partial g'} \frac{\partial g'}{\partial \epsilon} = \eta(x) \frac{\partial F}{\partial g} + \eta'(x) \frac{\partial F}{\partial g'} \end{displaymath}

\begin{displaymath} \frac{\partial J}{\partial \epsilon} = \int_a^b \left[\eta(x) \frac{\partial F}{\partial g} + \eta'(x) \frac{\partial F}{\partial g'} \right] dx. \end{displaymath}

$\epsilon = 0$ $g_\epsilon = f$, f 가 J
 $J'(0) = 0$,

\begin{displaymath} J'(0) = \int_a^b \left[\eta(x) \frac{\partial F}{\partial f} + \eta'(x) \frac{\partial F}{\partial f'} \right] dx = 0. \end{displaymath}

\begin{displaymath} 0 = \int_a^b \left[\frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'} \right] \eta(x) dx + \left[\eta(x) \frac{\partial F}{\partial f'} \right]_a^b. \end{displaymath}

η ,

\begin{displaymath} 0 = \int_a^b \left[\frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'} \right] \eta(x) dx. \end{displaymath}

\begin{displaymath} 0 = \frac{\partial F}{\partial f} - \frac{d}{dx} \frac{\partial F}{\partial f'}. \end{displaymath}

가

2 $\left. f \right|_a^b = \left. y \right|_a^b$ 가
가 L

\begin{displaymath} L[f] = \int_a^b \sqrt{1 + f'(x)^2} dx \end{displaymath}

$f(a) = y_a, f(b) = y_b$

f

\begin{displaymath} 0 = -\frac{d}{dx} \frac{\partial L}{\partial f} \left|_a^b \right. = -\frac{d}{dx} \sqrt{1 + f'(x)^2} \left|_a^b \right. \end{displaymath}

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\begin{displaymath} \begin{matrix} 0 &=& \frac{d}{dx} \frac{\partial }{\partial f'} \sqrt{1 + f' \left(x\right)^2} \\ &=& \frac{d}{dx} \frac{f' \left(x\right)}{\sqrt{1 + f' \left(x\right)^2}} \end{matrix} \end{displaymath}
```

0

,

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\begin{displaymath} \frac{f' \left(x\right)}{\sqrt{1 + f' \left(x\right)^2}} = k \end{displaymath}
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가 , $f' \left(x\right) = k$

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\begin{displaymath} f' \left(x\right) = C \end{displaymath}
```

가

 $f \left(x\right) = Cx + D$

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